HW 4 2012

Saturday, August 18, 2012
9:21 PM

26 a

$$900 A_{91} = 180v + 288v^2 + 216v^3 + 216v^4$$

$$= 816.01$$

$$A_{91} = \frac{816.01}{900} = \boxed{0.90668}$$

b $$900 A^{1}_{91:\overline{3|}} = 180v + 288v^2 + 216v^3$$

$$A^{1}_{91:\overline{3|}} = \frac{631.3723}{900} = \boxed{0.70152}$$

c $${}_3E_{91} = v^3\,{}_3p_{91} = \left(\frac{1}{1.04}\right)^3 \frac{216}{900} = \boxed{0.21336}$$

d

$$A_{91:\overline{3|}} = A^{1}_{91:\overline{3|}} + {}_3E_{91} =$$

$$0.70152 + 0.21336$$
$$= 0.91488$$

$$1000 A_{91:\overline{3|}} = \boxed{914.88}$$

e $${}_{2|}A_{91} = {}_2E_{91} \cdot A_{93} =$$

$$\left(\frac{1}{1.04}\right)^2 \left(\frac{432}{900}\right)\left(\frac{216v + 216v^2}{432}\right)$$

$$= \boxed{0.41851}$$

f $$\mathrm{Var}[Z] = {}^2A_{91} - (A_{91})^2$$

(f) $Var[Z] = {}^2A_{91} - (A_{91})^2$

$$900\ {}^2A_{91} = 180v^2 + 288v^4 + 216v^6 + 216v^8$$

$${}^2A_{91} = 0.82349$$

$$Var[Z] = {}^2A_{91} - (A_{91})^2 =$$

$$0.82349 - (0.90668)^2 = \boxed{0.00142}$$

(g) $$900\ A_{91}^{(4)} = \frac{180}{4}\left(v^{1/4} + v^{1/2} + v^{3/4} + v^1\right)$$

$$+ \frac{288}{4}\left(v^{5/4} + v^{6/4} + v^{7/4} + v^2\right)$$

$$+ \frac{216}{4}\left(v^{9/4} + v^{10/4} + v^{11/4} + v^3\right)$$

$$+ \frac{216}{4}\left(v^{13/4} + v^{14/4} + v^{15/4} + v^{16/4}\right)$$

$$= 45\left(\frac{v^{1/4} - v^{5/4}}{1 - v^{1/4}}\right) +$$

$$72\left(\frac{v^{5/4} - v^{9/4}}{1 - v^{1/4}}\right)$$

$$+ 54\left(\frac{v^{9/4} - v^{17/4}}{1 - v^{1/4}}\right)$$

$$= 828.15$$

$$A_{91}^{(4)} = \frac{828.15}{900} = \boxed{0.92017}$$

(h) $$432\ A^{(12)} = \frac{216}{} \left(v^{1/12} + v^{2/12} + \dots + v^{12/12}\right)$$

(h) $432 A_{93}^{(12)} = \frac{216}{12}\left(v^{1/12} + v^{2/12} + \cdots + v^{12/12}\right)$

$+ \frac{216}{12}\left(v^{13/12} + \cdots + v^{24/12}\right)$

$= 18\left(\frac{v^{1/12} - v^{25/12}}{1 - v^{1/12}}\right) = 414.8125$

$1000 A_{93}^{(12)} = 1000\left(\frac{414.8125}{432}\right)$

$= \boxed{960.21411}$

(i) $900\,(IA)_{91} = (1)(180)(v) +$

$(2)(288)v^2 + (3)(216)v^3 + (4)(216)v^4$

$(IA)_{91} = \boxed{2.24471}$

(j) $900\,(IA)^{1}_{91:\overline{3}|} = (1)(180)(v) +$

$(2)(288)v^2 + (3)(216)v^3$

$(IA)^{1}_{91:\overline{3}|} = \boxed{1.42410}$

(27) (a) $A_{50} = \boxed{0.24905}$ straight from Table

(b) $A^{1}_{50:\overline{20}|} = A_{50} - {}_{20}E_{50}\,A_{70}$

$= 0.24905 - (0.23047)(0.51495)$

$= \boxed{0.13037}$

$$= \boxed{0.13037}$$

(c) $A_{50:\overline{20|}} = A^{1}_{50:\overline{20|}} + {}_{20}E_{50}$

$$= 0.13037 + 0.23047$$

$$= \boxed{0.36084}$$

(d) ${}_{20|}A_{50} = {}_{20}E_{50}\, A_{70} =$

$$(0.23047)(0.51495) = \boxed{0.11868}$$

(e) $A^{1}_{50:\overline{35|}} = A_{50} - {}_{35}E_{50}\, A_{85}$

$$= A_{50} - {}_{20}E_{50}\ {}_{10}E_{70}\ {}_{5}E_{80}\ A_{85}$$

$$= (0.24905) - (0.23047)(0.33037)(0.45019)(0.73407)$$

$$= \boxed{0.22388}$$

(f) $Var[Z] = {}^{2}A_{50} - (A_{50})^2$

$$= 0.09476 - (0.24905)^2$$

$$= \boxed{0.03273}$$

(g) $Var[Z] = {}^{2}A^{1}_{50:\overline{20|}} - (A^{1}_{50:\overline{20|}})^2$

$A^{1}_{50:\overline{20|}} = 0.13037$ from part (b)

$${}^{2}A^{1}_{50:\overline{20|}} = {}^{2}A_{50} - v^{40}\, {}_{20}p_x\, {}^{2}A_{70}$$

$${}^2A^{1}_{50:\overline{20|}} = {}^2A_{50} - v^{40}\,{}_{20}p_x\,{}^2A_{70}$$

$$= 0.09476 - (1.06)^{40}\frac{6{,}616{,}155}{8{,}950{,}901}(0.30642)$$

$$= 0.07274$$

$$Var = 0.07274 - (0.13037)^2$$

$$= \boxed{0.05574}$$

(h) $${}_{13}E_{40} = v^{13}\,{}_{13}p_{40}$$

$$= \left(\frac{1}{1.06}\right)^{13}\frac{l_{53}}{l_{40}}$$

$$= \left(\frac{1}{1.06}\right)^{13}\frac{8{,}779{,}128}{9{,}313{,}166}$$

$$= \boxed{0.44195}$$

(28) $$A_x = vq_x + vp_x A_{x+1}$$

$$\Rightarrow 0.5 = vq_x + vp_x(0.50617)$$

$${}_{1|}A_x = vp_x A_{x+1}$$

$$\Rightarrow 0.410 = vp_x(0.50617)$$

$$vp_x = \frac{0.410}{0.50617} = 0.81$$

$$0.5 = vq_x + 0.81(0.50617)$$

$\Rightarrow vq_x = 0.09$

$vp_x = v(1-q_x) = v - vq_x = v - 0.09 = 0.81$

$\therefore v = 0.9 \qquad i = \frac{1}{v} - 1 = \boxed{0.111\bar{1}}$

$vq_x = 0.09$

$.9\, q_x = 0.09 \Rightarrow \boxed{q_x = 0.10}$

(29) $Var[Z] = {}^2A_{91} - (A_{91})^2$

$0.00706 = 0.54696 - (A_{91})^2$

$\Rightarrow A_{91} = \sqrt{0.54696 - 0.00706} = 0.73479$

$A_{90} = vq_x + vp_x\, A_{91}$

$= \frac{0.08}{1.08} + \frac{0.92}{1.08}(0.73479)$

$= \boxed{0.70000}$

(30) $A_{70} = vq_{70} + vp_{70}\, A_{71}$

$= \frac{1}{1.06}(2)(.03318) +$

$\frac{1}{1.06}\left(1-(2)(0.03318)\right)(0.53026)$

$= 0.529653$

$100{,}000\, A \quad = \boxed{52{,}965.3}$

$100{,}000\, A_{70} = \boxed{52{,}965.3}$

(31) (a)

$1000\,\bar{A}_{50} = 1000\,\frac{i}{\delta}\,A_{50} =$

$(1.02971)(249.05) = \boxed{256.45}$

(b) $1000\,\bar{A}^{1}_{50:\overline{20|}} = 1000\,\frac{i}{\delta}\,A^{1}_{50:\overline{20|}}$

$= 1000\,(1.02971)(0.13037) = \boxed{134.24}$

From (30) (b)

(c) $1000\,\bar{A}_{50:\overline{20|}} = 1000\,\bar{A}^{1}_{50:\overline{20|}} +$

$1000\,{}_{20}E_{50}$

From (b)

$= 134.24 + 230.47 = \boxed{364.71}$

(d) $1000\,{}_{20|}\bar{A}_{50} = {}_{20}E_{50}\,(1000\,\bar{A}_{70})$

$= {}_{20}E_{50}\left(\frac{i}{\delta}\right)(1000\,A_{70})$

$= (0.23047)(1.02971)(514.95) = \boxed{122.21}$

(e) $1000\,A^{(4)}_{50} = 1000\,\frac{i}{i^{(4)}}\,A_{50}$

$= (1.02223)(249.05) = \boxed{254.59}$

(32) (a) $1000\,\bar{A}_{50} = 1000\,A_{50}\,(1.06)^{1/2}$

$= 249.05\,(1.06)^{1/2} = \boxed{256.41}$

(b) $1000\,\bar{A}^{\,1}_{50:\overline{20}|} = 1000\,A^{\,1}_{50:\overline{20}|}\,(1.06)^{1/2}$

$= 130.37\,(1.06)^{1/2} = \boxed{134.22}$

FROM (30)(b)

(c) $1000\,\bar{A}_{50:\overline{20}|} =$

$1000\,\bar{A}^{\,1}_{50:\overline{20}|} + {}_{20}E_{50} =$

$134.22 + 230.47 = \boxed{364.69}$

FROM (b)

(d) $1000\,{}_{20|}\bar{A}_{50} = 1000\,{}_{20|}A_{50}\,(1.06)^{1/2}$

$= 1000\,{}_{20}E_{50}\,A_{70}\,(1.06)^{1/2}$

$= (0.23047)(514.95)(1.06)^{1/2}$

$= \boxed{122.19}$

(e) $1000\,A^{(4)}_{50} = 1000\,A_{50}\,(1.06)^{3/8}$

$= 249.05\,(1.06)^{3/8} = \boxed{254.55}$

(33) (a) $A^{\,1}_{54:\overline{3}|}$

$= v\,q_{[54]} + v^2\,(1 - q_{[54]})(q_{[54]+1})$

$$+ v^3 (1-q_{[54]})(1-q_{[54]+1})(q_{[54]+2})$$

$$= \left(\frac{1}{1.06}\right)(.04) + \left(\frac{1}{1.06}\right)^2 (.96)(.055) +$$

$$\left(\frac{1}{1.06}\right)^3 (.96)(.945)(0.076)$$

$$= \boxed{0.14262}$$

(b) $A_{[54]:\overline{3|}} = A^{1}_{[54]:\overline{3|}} + {}_3E_{54}$

$$0.14262 + v^3(1-q_{[54]})(1-q_{[54]+1})(1-q_{[54]+2})$$

$$= 0.14262 + \left(\frac{1}{1.06}\right)^3 (.96)(.945)(.924)$$

$$= \boxed{0.84643}$$

(34) $A_{x:\overline{3|}} = v\, q_x + v^2 p_x\, q_{x+1} + v^3 p_x \cdot p_{x+1}$

$$= \left(\frac{1}{1.1}\right)(.2) + \left(\frac{1}{1.1}\right)^2 (.8)(.4)$$

$$+ \left(\frac{1}{1.1}\right)^3 (.8)(.6)$$

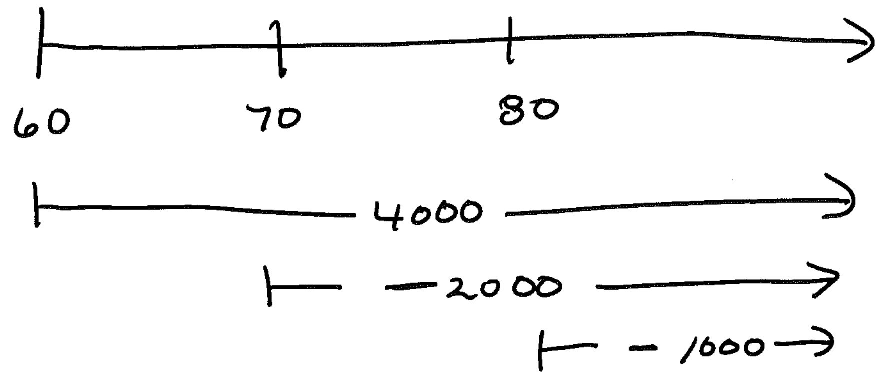
$$= \boxed{0.80691}$$

(35) $EPV = \int_0^{\infty} b_t \cdot v^t \cdot {}_tp_x\, \mu_{x+t}\, dt$

$$= \int_0^{\infty} (1.06)^t \left(\frac{1}{1.06}\right)^t {}_tp_x\, \mu_{x+t}\, dt$$

$$= \int_0^\infty {}_tp_x \, \mu_{x+t} \, dt = {}_\infty q_x = \boxed{1}$$

36



$$EPV = 4000 A_{60} - 2000 \, {}_{10|}A_{60} - 1000 \, {}_{20|}A_{60}$$

$$= 4000 A_{60} - 2000 \, {}_{10}E_{60} A_{70} - 1000 \, {}_{20}E_{60} A_{80}$$

$$= 4(369.13) - 2(.45120)(574.95) - (1)(0.14906)(665.75)$$

$$= \boxed{912.59}$$

37

$$E(Z) = 5000 A_{45} - 3000 \, {}_{20}E_{45} A_{65}$$

$$= 5000(.20120) - 3000(.25634)(.43980)$$

$$= 667.78500$$

$$E[Z^2] = E\left[\left(5000 v^{K+1} - 3000 v^{K+1} (I = K > 20)\right)^2\right]$$

$$= E\Big[(5000)^2 v^{2K+2} -$$

$$(3000)(5000) v^{K+1} v^{K+1} (I = K > 20)$$

$$+ 3000^2 v^{2K+2} (I = K > 20)\Big]$$

$$= (5000)^2 \; {}^2A_{45} - (2000)(3000) v^{20} {}_{20}E_{45} \; {}^2A_{65}$$

$$= (5000)^2 (.06802) - (2000)(3000)\left(\frac{1}{1.06}\right)^{20}(.25634)(.23603)$$

$$= 1,587,307.53$$

$$Var = 1,587,307.53 - (667.785)^2 = 1,141,370.72$$

(38) $EPV = A_{80}$ at i^* where

$$i^* = \frac{1+i}{1+j} - 1 = \frac{1.113}{1.05} - 1 = 0.06$$

so we can just use A_{80} from table

$$= \boxed{0.66575}$$

(39) $$A^{SMOKERS}_{\overset{1}{x}:\overline{2|}} = \left(\frac{1}{1.02}\right)(0.10) + \left(\frac{1}{1.02}\right)^2(0.9)(0.2)$$

$$= 0.27105$$

$$A^{NONSMOKERS}_{\overset{1}{x}:\overline{2|}} = \left(\frac{1}{1.02}\right)(0.05) + \left(\frac{1}{1.02}\right)^2(0.95)(0.10)$$

$$= 0.14033$$

$$10,000 \, A_{\overset{1}{x}:\overline{2|}} = \qquad -$$

$10{,}000\, A^{1}_{x:\overline{2|}} =$

$10{,}000\left[(.25)(0.27105) + (.75)(0.14033)\right]$

$= \boxed{1730.10}$

(40) The bulbs that fail during the first year is

$(10{,}000)(0.1) = 1000$

At the beginning of the second year, there are 9000 1 year old bulbs and 1000 new bulbs.

So bulbs that fail are

$(9000)(0.3) + (1000)(0.1) = 2800$

At beginning of year 3, there are

6300 2 year old bulbs
900 1 year old bulbs
2800 new bulbs

Failures during the third year are

$(6300)(0.5) + (900)(0.3) + (2800)(0.1)$

$= 3700$

$EPV = v(1000) + v^2(2800) + v^3(3700)$

$$= \underline{|6688.26|}$$

(41) $APV_{80}^{q=0.2} = 13 = v\,q_{80}(20) + v\,p_{80}\,(APV_{81})$

$$= \frac{(20)(.2)}{1.06} + \frac{.8}{1.06}\left(APV_{81}\right)$$

$$\therefore APV_{81} = \frac{13 - \frac{20(.2)}{1.06}}{\frac{.8}{1.06}} = 12.225$$

$$APV_{80}^{q=0.1} = v\,q_{80}(20) + v\,p_{80}\left(APV_{81}\right)$$

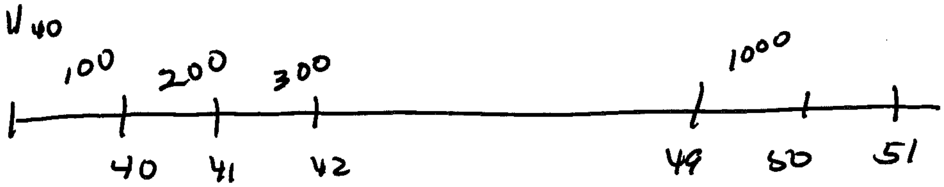
$$= \frac{1}{1.06}(.1)(20) + \frac{1}{1.06}(.9)(12.225)$$

$$= \boxed{12.27}$$

(42) $APV_{40} = \underset{(a)}{v\,p_{40}\,APV_{41}} + \underset{(b)}{100{,}000\,A^1_{40:\overline{10}|}}$

$$- \underset{(c)}{1{,}000{,}000\;{}_{10}E_{40}\,v\,q_{50}}$$

APV_{40}



(a) ⇒		100	200	900	1000
(b) ⇒	100	100	100	100	
(c) ⇒	—	—	—	—	−1000
TOTAL	100	200	300	1000	0

$$= v\,p_{40}(16{,}736) + 100{,}000\left(A_{40} - {}_{10}E_{40} \cdot A_{50}\right)$$

$$- 1{,}000{,}000 \, {}_{10}E_{40} \, v \, q_{50}$$

$$= \left(\frac{1}{1.06}\right)(0.99722)(16{,}736) +$$

$$100{,}000 \left[0.16132 - 0.53667(0.24905)\right]$$

$$- 1{,}000{,}000 \, (0.53667)\left(\frac{1}{1.06}\right)(0.00592)$$

$$= \boxed{15{,}513.77}$$

(43) $E[Z] = 300 v q_x + 350 v^2 p_x q_{x+1}$

$$+ 400 v^3 p_x p_{x+1} q_{x+2}$$

$$= \frac{300}{1.06}(.02) + \frac{(350)(.98)(.04)}{(1.06)^2}$$

$$+ \frac{400(.98)(.96)(.06)}{(1.06)^3}$$

$$= 36.82906$$

$E[Z^2] = (300)^2 v^2 q_x + (350)^2 v^4 p_x q_{x+1}$

$$+ (400)^2 v^6 p_x \cdot p_{x+1} \cdot q_{x+2}$$

$$= \frac{(300)^2(0.02)}{(1.06)^2} + \frac{(350)^2(.98)(.04)}{(1.06)^2}$$

$$+ \frac{(400)^2(.98)(.96)(.06)}{(1.06)^4}$$

$$= 11{,}722.60538$$

$\text{Var} = E[Z^2] - (E[Z])^2$

$$Var = E[Z^2] - (E[Z])^2$$

$$= 11{,}722.60638 - (36.82906)^2$$

$$= \boxed{10{,}417}$$

44 $E(Z) = 10{,}000 A_{65} = 4398.0$

$$Var(Z) = (10{,}000)^2 \left({}^2A_{65} - (A_{65})^2 \right)$$

$$= (10{,}000)^2 \left[.23603 - (.4398)^2 \right]$$

$$= 4{,}260{,}596$$

$$E(Port) = 100\ E(Z) = 439{,}800$$

$$Var(Port) = 100\ Var(Z) = 426{,}059{,}600$$

$$Amount = E(Port) + 1.282 \sqrt{Var(Port)}$$

$$= 439{,}800 + (1.282)\left(\sqrt{426{,}059{,}600}\right)$$

$$= 466{,}262$$

45 $E(Z) = 100{,}000 \left[A_{70} - {}_{20}E_{70} A_{90} \right]$

$$= 100{,}000 \left[0.51495 - (0.04988)(.79346) \right]$$

$$= 47{,}537.22$$

$$Var(Z) = (100{,}000)^2 \left[{}^2A^{1}_{70:\overline{20|}} - \left(A^{1}_{70:\overline{20|}} \right)^2 \right]$$

$$Var(z) = (100{,}000)^2\left[{}^2A^{\ \ 1}_{70:\overline{20|}} - \left(A^{\ \ 1}_{70:\overline{20|}}\right)^2\right]$$

$$= (100{,}000)^2\left[{}^2A^{\ \ 1}_{70:20} - {}_{20}E_{70}\left(\frac{1}{1.06}\right)^{20}\ {}^2A_{90} - (0.4753722)^2\right]$$

$$= (100{,}000)^2\left[.30642 - (0.04988)\left(\frac{1}{1.06}\right)^{20}(.64496) - (0.4753722)^2\right]$$

$$= (100{,}000)^2(0.070410325)$$

$$E(PORT) = (400)(47{,}537.22) = 19{,}014{,}888$$

$$S.D(PORT) = \sqrt{400\ Var(z)}$$

$$= \sqrt{400(100{,}000)^2(0.070410325}$$

$$= 530{,}698.88$$

$$E(PORT) + \gamma\ SD(PORT) = 20{,}000{,}000$$

$$19{,}014{,}888 + \gamma\ (530{,}698.88) = 20{,}000{,}000$$

$$\gamma = \frac{20{,}000{,}000 - 19{,}014{,}888}{530{,}698.88}$$

$$= 1.86$$

$$\Rightarrow \text{PROB} = 0.9686$$

46 $E[z] = 305.14$

$$\Pr(Z < 306.14) = \Pr\left[1000\,v^{K+1} < 306.14\right]$$

$$= \Pr\left(v^{K+1} < 0.30614\right)$$

$$= \Pr\left[K+1 > \frac{\ln(0.30614)}{\ln\left(\frac{1}{1.06}\right)}\right]$$

$$= \Pr\left[K+1 > 20.37\right]$$

$$= \Pr\left[K > 19.37\right] = {}_{20}p_{55}$$

$$= \frac{l_{75}}{l_{55}} = 0.62448$$

47 (a)

$$(IA)^1_{x:\overline{n}|} = v q_x + 2v^2 p_x q_{x+1} + 3\,{}_2p_x q_{x+2} + \cdots + n v^n\,{}_{n-1}p_x q_{x+n-1}$$

$$= v q_x + v p_x \left[2v q_{x+1} + 3 p_{x+1} q_{x+2} + \cdots + n v^{n-1}\,{}_{n-2}p_{x+1} q_{x+n-1}\right]$$

$$= v q_x + v p_x \left[\underline{v q_{x+1} + 2 p_{x+1} q_{x+2} + \cdots + (n-1) v^{n-1}\,{}_{n-2}p_{x+1} q_{x+n-1}}\right.$$

$$\left. + v q_{x+1} + v^2 p_{x+1} q_{x+2} + \cdots + v^{n-1}\,{}_{n-2}p_{x+1} q_{x+n-1}\right]$$

$$= v q_x + v p_x \left[(IA)^1_{x+1:\overline{n-1}|} + A^1_{x+1:\overline{n-1}|}\right]$$

$0 \quad \ldots \; \ldots)_{x+1:\overline{n-1|}} \quad \ldots_{x+1:\overline{n-1|}})$

(c) If $n \Rightarrow \infty$ then

$$(IA)_x = vq_x + vp_x\left[(IA)_x + A_x\right]$$

$$(IA)_{50} = vq_{50} + vp_{50}\left[(IA)_{51} + A_{51}\right]$$

$$vq_{50} = A^{1}_{50:\overline{1|}} = 0.00558$$

so

$$4.99675 = 0.00558 +$$

$$v(1-q_{50})\left[(IA)_{51} + 0.24905\right]$$

$$4.99675 = 0.00558 + \left(\frac{1}{1.06} - 0.00558\right)$$

$$\left[(IA)_{51} + 0.24905\right]$$

$$(IA)_{51} = \boxed{5.07307}$$

(48) $$A_{50:\overline{20|}} = A^{1}_{50:\overline{20|}} + {}_{20}E_{50}$$

$$\therefore {}_{20}E_{50} = 0.42247 - 0.14996$$

$$= 0.27251$$

$$A_{50} = A^{1}_{50:\overline{20|}} + {}_{20}E_{50}\, A_{70}$$

$$\therefore A_{70} = A_{50} - A^{1}_{50:\overline{20|}}$$

$\cdot 7 \cdot 70 \quad \dfrac{\qquad}{{}_{20}E_{50}} =$

$$\frac{0.31266 - 0.14996}{0.27251}$$

$$= \boxed{0.59704}$$

49

$$Var[Y] = {}^2A^{1}_{x:\overline{n}|} - (A^{1}_{x:\overline{n}|})^2$$

$$Var[X] = {}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2 = 0.0052$$

$$A^{1}_{x:\overline{n}|} = E(Y) = 0.04$$

$$A_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + {}_nE_x = 0.04 + v^n {}_np_x$$

$$= 0.04 + (0.3)(0.8) = 0.28$$

$$0.0052 = {}^2A_{x:\overline{n}|} - (0.28)^2$$

$${}^2A_{x:\overline{n}|} = 0.0836$$

$${}^2A_{x:\overline{n}|} = {}^2A^{1}_{x:\overline{n}|} + v^{2n} {}_np_x$$

$$0.0836 = {}^2A^{1}_{x:\overline{n}|} + (.3)^2(.8)$$

$${}^2A^{1}_{x:\overline{n}|} = 0.0116$$

$$Var[Y] = {}^2A^{1}_{x:\overline{n}|} - (A^{1}_{x:\overline{n}|})^2$$

$$= 0.0116 - (0.04)^2$$

$= \boxed{0.01}$